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DETERMINISTIC TOLERANCE ANALYSIS AND TOLERANCE REDESIGN FOR 3D NON-RIGID ASSEMBLY

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A B S T R A C T

Tolerance analysis and redesign are essential activities performed by the designers to ensure the quality and functionalities of the individual parts and assembly. During the manufacturing and assembly stages, the individual parts deviates from the ideal dimensions due to the variations from manufacturing and assembly process. These variations affect the overall quality and functionality of the assembly. In this paper, a deterministic method to performing tolerance analysis and redesign of deformed non-rigid parts in an assembly is presented. Using the deformation gradient models, the accumulation of the different type of deformation during the assembly process is concluded and integrated into the unified Jacobian-Torsor model to obtain a deformation mathematical model for the assembly. The deformed unified Jacobian-Torsor model is used to perform tolerance analysis on the deformed assembly taking into consideration the impact of both manufacturing variations and deformation variations on the final design requirements. The deterministic approach to perform tolerance redesign of the deformed non-rigid assembly is later presented using the deformed unified Jacobian-Torsor model.

Keywords: Non-Rigid Assembly, Deformed Unified Jacobian-Torsor Model, Tolerance Redesign.

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1. INTRODUCTION

The parts manufactured for mechanical assemblies have some form of variations which often accumulate and propagate, therefore affecting the overall assembly dimension and function (Chen et al., 2014).

Tolerance analysis defines a procedure to estimate the resultant variation of the assembly dimension, given the tolerances associated with individual dimensions and the functional relationship between the individual dimensions and assembly dimensions (Singh et al., 2009).

Over the years, mathematical models used to form tolerance analysis include Vector loop model (Chase,1999), Torsor model (Takahashi et al., 2014), Jacobian model (Corrado and Polini, 2017), unified Jacobian-Torsor model (Desrochers et al., 2003), Topological and technological related surface (TTRS) model (Ameta et al., 2011), T-map model (Mansuy et al., 2013) and Homogeneous transformation matrix model (HTM) (Whitney, 2004).

In the early years, researchers assumed that the parts and the assembly are rigid and cannot experience some type of deformation, however some researchers have studied the significance of deformation of assembly parts on the functionality and quality of the assembly. Elastic deformation has been integrated into four models for tolerancing mechanisms (Samper and Giordano, 1998). Thermos-mechanical strains deformation has been modeled using finite element analysis and incorporated into tolerance analysis (Pierre et al., 2009). Thermal and inertia deformation of assembly have been studied for optimal tolerance design (Jayapakash et al., 2014). Thermal expansion deformation was integrated into tolerance allocation of a non-rigid assembly (Benichou and Aselmetti, 2011). A tolerance analysis method considering the influence of both internal and external forces on deformed compliant parts of an assembly has been demonstrated (Govindrarajalu et al., 2012). The precision analysis of the mechanical assembly subjected to deformation during working operation has been studied (Jin et al., 2018). Ajani and Lu (2021) performed assembly variation analysis of the non-rigid assembly subjected to joining forces during the assembly process.
In the literature, the unified Jacobian-Torsor (J-T) model has been successfully used to perform tolerance analysis for deformed assemblies, whiles using finite element model to simulate the deformation. The effect of part deformation during assembly operation was studied using the unfied J-T model (Zhang et al., 2011). Also, unified J-T model was used to increase assembly accuracy with part deformation (Shen et al., 2015). Deformation variations from weight of part, change in part temperature and external loads were used to predict the total assembly error (Ting et al., 2016).

After performing tolerance analysis, it was observed that the non-rigid assembly's accuracy deviated from the designer functional requirement due to variations. In attempt to rectify the deviation from the required functional requirement, tolerance redesign is performed to readjust the initially assigned

IJAMSR

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part tolerances. Ghie et al. (2007) presented an approach based on the unified J-T model to perform tolerance analysis and tolerance redesign for the rigid assembly, whiles not considering the deformation variations.

The purpose of this work is to present a mathematical model to perform tolerance analysis and tolerance redesign for non-rigid assembly undergoing deformation during the assembly process. In place of the finite element model, the deformation gradient model is used to model the accumulated deformation of the non-rigid assembly. A deformed unified J-T model is used to perform tolerance analysis on the non-rigid assembly, and to redesign the initial assigned tolerances which did not conform to the quality and functionality of the non-rigid assembly.

2. MATHEMATICAL MODELS

2.1 Unified Jacobian-Torsor Model

The Torsor model and the Jacobian model are combined to obtained the unified Jacobian-Torsor (J-T) model. The unified J-T model harmonized the advantages of good tolerance propagation of the Jacobian matrix with the good tolerance representation of the Torsor model (Desrochers et al., 2003). The unified Jacobian-Torsor model can be expressed as:

$$[FR] = [j] \cdot [FE] \qquad (1)$$

$$\begin{bmatrix} [^{-}u, u^{+}] \\ [^{-}v, v^{+}] \\ [^{-}w, w^{+}] \\ [^{-}\theta, \theta^{+}] \\ [^{-}\phi, \phi^{+}] \\ [^{-}\varphi, \varphi^{+}] \end{bmatrix}_{FR} = \left[[j]_{FE1} \cdots [j]_{FEn} \right] \times \left[\begin{bmatrix} [^{-}u, u^{+}] \\ [^{-}v, v^{+}] \\ [^{-}w, w^{+}] \\ [^{-}\theta, \theta^{+}] \\ [^{-}\phi, \phi^{+}] \\ [^{-}\varphi, \varphi^{+}] \end{bmatrix}_{FE1} \cdots \begin{bmatrix} [^{-}u, u^{+}] \\ [^{-}v, v^{+}] \\ [^{-}w, w^{+}] \\ [^{-}\theta, \theta^{+}] \\ [^{-}\phi, \phi^{+}] \\ [^{-}\varphi, \varphi^{+}] \end{bmatrix}_{FEn} \right]^{T} \qquad (2)$$

Where FR is the functional requirement; $[j]$ is the Jacobian matrix; FE is the functional elements; u, v, w are the linear displacements ; θ, ϕ, φ are the angular displacements; n represents the number of the functional elements which are defined in Eq. (4). The Jacobian matrix at the i-th FE can be expressed as:

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$$[j]_{FEi} = [j]_0^i = \begin{bmatrix} \left[{}_{PT}\varpi_0^i\right]_{3\times3} & \vdots & \left[S_i^n\right]_{3\times3} \cdot \left[{}_{PT}\varpi_0^i\right]_{3\times3} \\ \cdots & \vdots & \cdots \\ [0]_{3\times3} & \vdots & \left[{}_{PT}\varpi_0^i\right]_{3\times3} \end{bmatrix}_{6\times6} \quad \text{———— (3)}$$

with

$$\left[{}_{PT}\varpi_0^i\right]_{3\times3} = \left[\varpi_0^i\right]_{3\times3} \cdot \left[\varpi_{PTi}\right]_{3\times3} \quad \text{———— (4)}$$

Where $\left[\varpi_0^i\right]$ describes the local orientation of the *i*-th coordinate frame relative to the 0-th coordinate frame located in the global frame; $\left[\varpi_{PTi}\right]$ projects the unit vectors on the local axes respectively for the tilted tolerance zone with respect to the direction of the tolerance analysis; $\left[{}_{PT}\varpi_0^i\right]$ is the projected orientation vectors; $\left[S_i^n\right]$ is the skew-symmetric matrix which represents the position vector between the *n*-th and *i*-th frame such that $\Delta\varepsilon_i^n = \Delta\varepsilon_n - \Delta\varepsilon_i$ with $\Delta u_i^n = \Delta u_n - \Delta u_i$, $\Delta v_i^n = \Delta v_n - \Delta v_i$, $\Delta w_i^n = \Delta w_n - \Delta w_i$. The skew-symmetric matrix can be expressed as:

$$\left[S_i^n\right]_{3\times3} = \begin{bmatrix} 0 & -\Delta w_i^n & \Delta v_i^n \\ \Delta w_i^n & 0 & -\Delta u_i^n \\ -\Delta v_i^n & \Delta u_i^n & 0 \end{bmatrix} \quad \text{———— (5)}$$

2.2 Deformation Gradient Model

During the assembly process of deformable parts (non-rigid, compliant, and flexible), dimensional and geometrical deviations should be analyzed from both the manufacturing process and the assembly process. Ajani and Lu (2022) presented the deformation gradient model to mathematically present the deformation variations between the deformed non-rigid part's nominal surface and an ideal surface in terms of position and orientation. The accumulated deformation variations, based on the assembly sequence, is also modeled by the deformation gradient model.

The deformation gradient model used to obtain the translational deformation, the rotational deformation and the strain deformation of the deformed surface can be expressed as:

$$\varepsilon_d = \varepsilon_{dT} + D \cdot \varepsilon \qquad (6)$$

Where ε_d is the deformed new location, ε_{dT} describes the translational deformation, D is the deformation gradient describing both rotational and strain deformation, and ε is the coordinates of the points to be deformed.

The rotational and strain deformation of the deformation gradient model considers mechanical extension or compression, mechanical shear and bending type of deformation. As such, the deformation gradient is expressed as:

$$D = I + \in + \omega \qquad (7)$$

$$\in = \begin{pmatrix} \in_{xx} & \in_{xy} & \in_{xz} \\ \in_{xy} & \in_{yy} & \in_{yz} \\ \in_{xz} & \in_{yz} & \in_{zz} \end{pmatrix}, \ \omega = \begin{pmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{pmatrix}$$

Where I is 3×3 identity matrix, $\in$ is the engineering strain tensor which is describing the strain deformation due to change in size and shape, and ω is the rotational tensor which is describing the rotational deformation due to the change in orientation of the non-rigid part.

For the non-rigid part experiencing different types of deformation in a specific sequence, the accumulated deformation can be expressed in terms of the accumulated deformation gradient. The mathematical expression for the accumulated deformation gradient D_A from successive deformations of $D_1, \ D_2, \ \ldots \ D_n$ can be modeled as:

$$D_A = D_n \cdot D_{n-1} \cdot D_{n-2} \cdot \ldots \cdot D_1 \qquad (8)$$

3. TOLERANCE ANALYSIS AND REDESIGN FOR DEFORMED NON-RIGID ASSEMBLY

3.1 Deformed Unified J-T Model for Deterministic Tolerance Analysis

The unified J-T model used for tolerance analysis estimates the accumulation of the assigned tolerances of parts to obtain the functional requirement of the rigid assembly. The model operates under the assumption that positions of parts in the assembly do not change due to the deformation, however, during the assembly process, these positions change due to the deformations. Using the deformation gradient model, the obtained deformed positions can be integrated into the rigid unified

IJAMSR

International Journal of
Advanced Multidisciplinary Scientific Research (IJAMSR) ISSN:2581-4281

J-T model to achieve a deformed unified J-T model for non-rigid assembly (Ajani and Lu, 2022). Fig. 1 shows the framework to obtain the functional requirement for tolerance analysis of the rigid and non-rigid assembly.

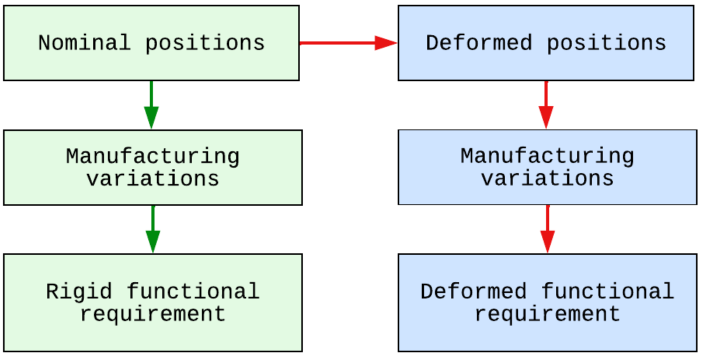

Fig. 1: Framework for Obtaining Rigid and Deformed Functional Requirements

The deformation gradient model is used to obtain the deformed positions from the nominal position of the non-rigid assembly parts which are subjected to some type of internal or (and) external forces. To describe the positional vectors of the nominal position and the deformed position, the deformation gradient model can be expressed:

$$\begin{bmatrix} u_D \\ v_D \\ w_D \end{bmatrix} = \begin{bmatrix} u_{dT} \\ v_{dT} \\ w_{dT} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} . \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \text{———} \quad (9)$$

Where $\begin{bmatrix} u_D & v_D & w_D \end{bmatrix}$ is the deformed position vector, $\begin{bmatrix} u_{dT} & v_{dT} & w_{dT} \end{bmatrix}$ is the translational deformation vector, D is the deformation gradient matrix and $\begin{bmatrix} u & v & w \end{bmatrix}$ is the nominal position vector. The position vector between the deformed frame and *i*-th frame is expressed as $\Delta\varepsilon_i^D = \Delta\varepsilon_D - \Delta\varepsilon_i$, such that $\Delta u_i^D = \Delta u_D - \Delta u_i$, $\Delta v_i^D = \Delta v_D - \Delta v_i$, $\Delta w_i^D = \Delta w_D - \Delta w_i$. The deformed skew-symmetric matrix describing the position vector between deformed and *i*-th frame is expressed as:

$$\left[S_i^D \right]_{3\times3} = \begin{bmatrix} 0 & -\Delta w_i^D & \Delta v_i^D \\ \Delta w_i^D & 0 & -\Delta u_i^D \\ -\Delta v_i^D & \Delta u_i^D & 0 \end{bmatrix} \quad \text{———} \quad (10)$$

IJAMSR

International Journal of
Advanced Multidisciplinary Scientific Research (IJAMSR) ISSN:2581-4281

The deformed skew-symmetric matrix is integrated into the rigid unified J-T matrix (Eq. (3)) to obtain the deformed unified J-T matrix which can be expressed as:

$$[j_D]_{FEi} = [j_D]_0^i = \begin{bmatrix} \left[{}_{PT}\varpi_0^i \right]_{3\times3} & \vdots & \left[S_i^D \right]_{3\times3} \cdot \left[{}_{PT}\varpi_0^i \right]_{3\times3} \\ \cdots & \vdots & \cdots \\ [0]_{3\times3} & \vdots & \left[{}_{PT}\varpi_0^i \right]_{3\times3} \end{bmatrix}_{6\times6} \quad \text{\_\_\_\_\_\_\_} (11)$$

The general expression for the deformed unified J-T model can be written as:

$$[FR_D] = [j_D].[FE] \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (12)$$

Where $[FR_D]$ is the functional requirement of the assembly including part deformation, $[j_D]$ is the Jacobian matrix of the deformed parts and $[FE]$ is the functional elements. In terms of small linear and angular displacement torsor, the above expression can be described as:

$$\begin{bmatrix} [{}^{-}u, u^{+}]_D \\ [{}^{-}v, v^{+}]_D \\ [{}^{-}w, w^{+}]_D \\ [{}^{-}\theta, \theta^{+}]_D \\ [{}^{-}\phi, \phi^{+}]_D \\ [{}^{-}\varphi, \varphi^{+}]_D \end{bmatrix}_{FR_D} = \left[[j_D]_{FE1} \cdots [j_D]_{FEn} \right] \times \begin{bmatrix} \begin{bmatrix} [{}^{-}u, u^{+}] \\ [{}^{-}v, v^{+}] \\ [{}^{-}w, w^{+}] \\ [{}^{-}\theta, \theta^{+}] \\ [{}^{-}\phi, \phi^{+}] \\ [{}^{-}\varphi, \varphi^{+}] \end{bmatrix}_{FE1} \cdots \begin{bmatrix} [{}^{-}u, u^{+}] \\ [{}^{-}v, v^{+}] \\ [{}^{-}w, w^{+}] \\ [{}^{-}\theta, \theta^{+}] \\ [{}^{-}\phi, \phi^{+}] \\ [{}^{-}\varphi, \varphi^{+}] \end{bmatrix}_{FEn} \end{bmatrix}^T \quad \text{\_\_\_\_\_\_\_} (13)$$

More details about deformed unified J-T model for non-rigid assembly, can be referred to the literature (Ajani and Lu, 2023).

3.2 Deterministic Tolerance Redesign for The Non-Rigid Assembly

Tolerance redesign is performed on mechanical assemblies to confirm and adjust the initially assigned tolerances during the design stages. This procedure is essential, especially in mechanical assemblies with deformable parts, to obtain a more accurate tolerance for the functionality of the assembly. Referring to the deterministic approach for tolerance analysis for the non-rigid mechanical assembly shown in the Eq. (12), a tolerance reassigning method is therefore proposed to the tolerance redesign problem associated with deformed non-rigid assembly. The proposed Eq. (13) of the functional requirement of a non-rigid assembly with manufacturing variations and deformation variations FR_D, can be expressed as the contribution of each functional element (*FE*) in the assembly

IJAMSR

International Journal of
Advanced Multidisciplinary Scientific Research (IJAMSR) ISSN:2581-4281

to the total deformed functional requirement. As such, the total contribution of each FE to the FR_D is expressed as:

$$\begin{bmatrix} [{}^{-}u,u^{+}]_D \\ [{}^{-}v,v^{+}]_D \\ [{}^{-}w,w^{+}]_D \\ [{}^{-}\theta,\theta^{+}]_D \\ [{}^{-}\phi,\phi^{+}]_D \\ [{}^{-}\varphi,\varphi^{+}]_D \end{bmatrix}_{FR_D} = C_{DFE1} + \ldots + C_{DFEn} \quad \text{———} \quad (14)$$

where

$$C_{DFEk} = \begin{bmatrix} j_{D1} & j_{D2} & j_{D3} & j_{D4} & j_{D5} & j_{D6} \end{bmatrix}_{FEk} \bullet \begin{bmatrix} [{}^{-}u,u^{+}] \\ [{}^{-}v,v^{+}] \\ [{}^{-}w,w^{+}] \\ [{}^{-}\theta,\theta^{+}] \\ [{}^{-}\phi,\phi^{+}] \\ [{}^{-}\varphi,\varphi^{+}] \end{bmatrix}_{FEk} \quad \text{———} \quad (15)$$

C_{DFEk} is describing the contribution from the manufacturing and deformation variations of the FEk to the total FR_D. The percentage contribution of individual FE to the total deformed FR_D is computed using the expression:

$$\%C_{DFEk} = \frac{\left| {}^{+}C_{DFEk} - {}^{-}C_{DFEk} \right|}{\sum_{k=1}^{n} \left| {}^{+}C_{DFEk} - {}^{-}C_{DFEk} \right|} \quad \text{———} \quad (16)$$

$\%C_{DFEk}$ is the representation of the percentage contribution of the k-th deformed FE to the total FR_D in the direction of interest in the tolerance analysis, with $k = 1,2,\ldots,n$. The percentage contribution of the individual FE to the total FR_D is used to construct a percentage chart to enable the designer to reassign tolerances to deformed parts due to their levels of variation contributions. Using the percentage contribution chart, the designer can easily identify the critical tolerances of the

IJAMSR
International Journal of
Advanced Multidisciplinary Scientific Research (IJAMSR) ISSN:2581-4281

deformed parts and adjust them to ensure the desired design values are achieved.

The framework to perform the tolerance analysis and redesign of the deformed non-rigid assembly is shown in Fig. 2.

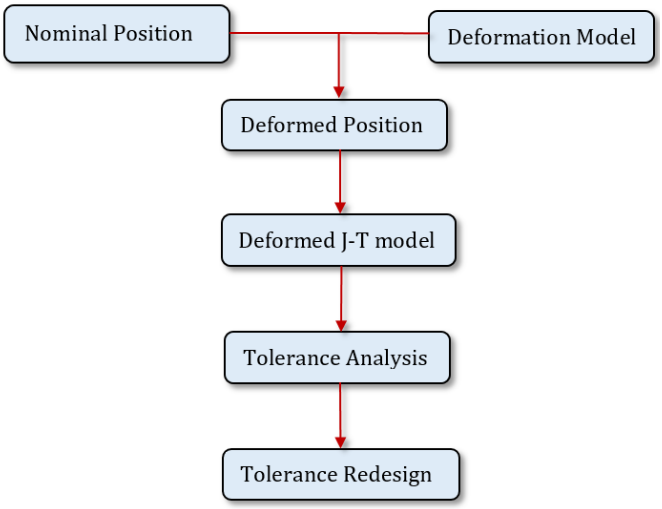

Fig. 2: Framework for Tolerance Analysis and Redesign of Deformed Assembly

CONCLUSION

This work presents a mathematical model to perform deterministic tolerance analysis and redesign on a deformed non-rigid assembly. The mathematical model takes into consideration the effect of accumulated deformation variations through the assembly process, the manufacturing variations, and readjust the initially assigned part tolerances to ensure assembly functionality and quality are enhanced.

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